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Review of the Doctoral Dissertation by mgr. inż. Hiren Kakkad

entitled

"Scattering Amplitudes in the Yang-Mills sector of Quantum Chromodynamics"

under supervision of dr hab. Piotr Kotko

The thesis manuscript is approximately 220 pages long. Some of the research results presented in the manuscript has been published in journal articles where Mr. Kakkad is a coauthor:

[1] H. Kakkad, P. Kotko, A. Stasto, "Exploring straight infinite Wilson lines in the self-dual and MHV Lagrangians", Phys. Rev. D 102 (2020) 9, 094026
[2] H. Kakkad, P. Kotko, A. Stasto, "A new Wilson line-based action for gluodynamics" JHEP 07 (2021) 187.
[3] H. Kakkad, P. Kotko, A. Stasto, "One-Loop effective action approach to quantum MHV theory" JHEP 11 (2022) 132.

Quantum Chromodynamics (QCD), the theory of strong interactions, has been very successful in describing strong scattering processes. One of the most significant observables that is frequently computed within the QCD framework is the cross section of hadronic collisions. Thanks to the factorization theorems, this cross section can be computed as a convolution of different types of Parton Distribution Functions (PDFs) (that are non-perturbative objects which define the structure of the hadrons) and partonic cross sections which can be calculated perturbatively. Partonic cross sections are defined as the square of the scattering amplitudes. The standard way to compute the scattering amplitudes are via well known Feynman diagrams. One major drawback in this approach is the fact that the number diagrams that need to be computed grows rapidly with the increasing number of external gluons and power of α_s. An alternative approach for the computation of the scattering amplitudes is to adopt maximally helicity violating (MHV) tree amplitudes while using the helicity spinors as the appropriate variables. For this case, one can derive the MHV action from the Yang-Mills action (the pure gluon sector of the QCD) by performing a specific canonical transformation. Analog of the Feynman rules in the MHV action corresponds to the MHV rules (also known as Cachazo, Svrcek, Witten rules) which simply states that off-shell MHV amplitudes can be used as vertices and one can get any tree-level pure gluonic amplitude by gluing these vertices together with a scalar propagator. It has been shown that this method requires considerably less number of diagrams for the computation of the scattering amplitudes. Following these ideas, the main topic of the thesis is to provide a new field theory action-based method to compute pure gluonic scattering amplitudes not only at tree level but also at one loop order.

Chapter 1 presents a concise introduction to the basic concepts that will be used in the rest of the thesis. After drawing a short outline of the thesis, the author first presents the basics of QCD. The main focus of the thesis is to compute the pure gluonic scattering amplitudes. Therefore, this chapter continues with the discussion of the scattering amplitudes in the Yang-Mills sector of QCD. After reviewing the computation of the scattering amplitudes from the Yang-Mills partition function, the author briefly reviews the Feynman diagram approach and points out the drawbacks of this approach when the number of external gluons increases. The chapter then continues with the introduction of the color decomposition and the spinor helicity formalism both of which will be used in the rest of the thesis. This chapter ends with the discussion of the MHV rules for the computation of the tree level pure gluonic amplitudes. The material presented in this chapter is very well balanced. It provides the key ideas and the basic calculational tools to be able to follow the work performed in the thesis without overwhelming the reader.

Chapter 2 starts with the derivation of the MHV action. The author shows that Yang-Mills action on the light cone can be transformed into MHV action after performing Mansfield transformations. The resulting action is shown to consist MHV vertices and a scalar propagator. Even though these are the known results in the literature, the author provides the details of the derivation in Appendix A1 and A2. The chapter continues with the discussion of the physical interpretation of the solutions of the transformations that generates the MHV action. First, it is shown that positive helicity field in the MHV action encodes the self-dual sector of the Yang-Mills theory and when written in coordinate space it can be interpreted as straight infinite Wilson line lying on the Self-Dual plane. Similar analysis is performed for the negative helicity field and it was shown that it can be interpreted as a similar Wilson line but with an insertion of the negative helicity field of the Yang-Mills action. Most of the work presented in this chapter is the subject of the Publication [1]. The details of the calculations of the presented material in this chapter are provided in Appendix A1, A2, A3 and A4.

After reinterpreting the solutions of the transformations that derives the MHV action in terms of the straight infinite Wilson lines, in Chapter 3 the derivation of a new Wilson line based action for computing pure gluonic amplitudes is presented. One of the most important aspects of the new Wilson line based action is the absence of any triple gluon interaction vertices that are present in the Yang-Mills action on the light cone which reduces dramatically the number diagrams for computing the scattering amplitudes. The details of the elimination of the three gluon vertices are presented in Appendix A5. The derivation of the new action is performed in two ways: first via direct canonical transformation from the Yang-Mills action and second via two consecutive canonical transformations from Yang-Mills action to MHV action and from MHV action to the new Wilson line based action. The physical interpretation of the two solutions of the transformations is also discussed in Chapter 3. Analogous to the interpretation of the presented in Chapter 2, the two solutions of the transformation of the new action are shown to be infinite Wilson lines of the fields that themselves are also Wilson lines spanning over both Self-Dual and Anti-Self-Dual planes. The computation of tree-level pure gluonic amplitudes with different helicity configurations (4 point to 8 point amplitudes) using the new Wilson line based action is also presented in Chapter 3. These results are also presented in publication [2]. Apart from these published results, in the rest of Chapter 3, two interesting observations, that are not yet published, are discussed: (i) the number of diagrams needed to compute helicity amplitudes using the Wilson line based action follows the number series known as "Delannoy numbers" and (ii) the vertices in the Wilson line based action can be interpreted as a twistor space prescription for computing gauge theory amplitudes.

The work performed in publications [1] and [2] and presented in Chapters 2 and 3, focuses on the computation of tree level amplitudes and the loop amplitudes were not considered. As a natural continuation of these studies, Chapter 4 is devoted to the computation of the one loop effective MHV action. As explained in detail in the text, it is not possible to compute the quantum corrections starting from the MHV action since there are missing loop contributions originating from the triple gluon vertex or the mixing of this vertex with the MHV vertices. Therefore, in order to compute the loop corrections to the MHV action, the author uses the one loop effective action of the Yang-Mills theory and systematically develops a method that can be used to compute the quantum corrections not only to the MHV action but also to the Wilson line based action developed in Chapter 3. For this purpose, the author first presents the derivation of the one loop effective action for the Yang-Mills theory in Chapter 4. Then, starting from the Yang-Mills one-loop partition function the author uses the Mansfield transformations to transform the classical fields of the Yang-Mills action to the fields of the MHV action which in turn provides the transformation of the Yang-Mills partition function to the MHV partition function. Then using this partition function one loop effective MHV action is derived. Finally, the author computes (all positive and and a single negative) four point one loop amplitudes using the one loop effective MHV action and show that these match with the known results. The details of these calculations are presented in Appendix A6, A7 and A8. The majority of the worked presented in Chapter 4 are tightly related with the results of the Publication [3].

In Chapter 5, the author extends the computation of the quantum corrections to the new Wilson line based action (throughout this chapter it is refereed to as Z-field action) derived in Chapter 3. For this purpose, the author adopts the same method (namely starting from the one loop effective action of the Yang-Mills theory) used in Chapter 4 to compute the one loop contributions to the Z-field action. However, the loop contributions to the action computed in this way still includes the Yang-Mills vertices. In order to overcome this issue, the author revisits the one loop effective action method. In this new approach, the author suggests to start from the Yang-Mills generating functional, perform the canonical transformations not only to the classical action but also to the source term that generates the loop contributions. In this way, the author gets access to the new nonlinear contributions in the loop correction which allows him to write the loop contributions in terms of the new (Z-field) vertices. Finally, the author computes all five possible 4 point one loop amplitudes by using the one loop effective Z-field action and show that this action is indeed one-loop complete with no missing one-loop contributions. Some details of the derivation and computations of the checks are presented in Appendix A9 and A10. The work presented in this Chapter has not

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been published yet, but in my opinion the novelty of the results and the completeness of the calculation guarantee a very good publication in a high impact journal.

Finally, Chapter 6 draws the overall conclusions on the work that is performed in the thesis. The text stresses very clearly all the steps to derive the Z-field action both at tree level and also the effective one loop Z-field action. The advantages of using the newly derived Z-field action has been discussed by mentioning the total number diagrams that should be computed. Moreover, the interesting observation of the number of diagrams needed to compute helicity amplitudes using the Z-field action follows the number series known as "Delannoy numbers" allow the author to predict number of diagrams needed to compute these amplitudes in the Z-field action and compare quantitatively with required number diagrams in the MHV action. Moreover, the physical interpretation of the solution of the new Z-field action is understood to be "Wilson lines of Wilson lines" as the degrees of freedom which suggests efficiency in the computation of the scattering amplitudes compared to Yang-Mills or the MHV actions. Regarding the quantum corrections, the author states the issues that had to be resolved and summarizes the solutions to overcome these issues in order to derive the one loop effective Z-field action. The conclusions are written in a very clear way which summarizes the work performed in the thesis. Moreover, in Chapter 7, the author also discusses possible future extensions of the results he obtained in the thesis. Additionally, reviews of some derivations along with the details of the various calculations are presented in the Appendix.

The content of the thesis is highly technical which is rather standard in theoretical physics. More precisely, the work performed in the thesis aims to understand the Yang-Mills sector (i.e. pure gluon sector) of the QCD by using the scattering amplitude method. The computations are performed in the Helicity Spinor Formalism which is known to be extremely efficient compared to the Feynman diagram approach especially for the cases when the number of external gluons is large. This formalism is very sophisticated especially when one wants to compute the loop corrections to the scattering amplitudes which constitutes almost half of the work presented in the thesis. The results presented in the thesis are novel and interesting, moreover significantly contribute to the development of the field by providing a new Wilson line based action (the Z-field action) both at classical and complete one loop level. The three published [1,2,3] and one under-preparation article each represent a significant calculation effort, and overall the amount of the novel scientific results presented in the thesis is, without a doubt, more than sufficient for a good doctoral thesis in theoretical physics. The work in the three published articles is performed by a group of three researchers including the author. This is a standard practice in the theoretical physics and completely fair given the complexity of the problems addressed in these publications. On the other hand, in the thesis, the author presents detailed derivations of the known results in the literature together with the details of the calculations of the novel problems addressed during his PhD. These show the independent role of the author in these published articles. Moreover, apart from the work presented in the thesis, the author also contributed to two research projects on the phenomenological aspects of the scattering amplitudes in QCD which resulted in the following two publications:

- H. Kakkad, A. K. Kohara, P. Kotko, " Evolution equation for elastic scattering of hadrons", Eur. Phys. J. C. 82 (2022) 9, 830.

- M. A. Al-Mashad, A. van Hameren, H. Kakkad, P. Kotko, K. Kutak, P. van Mechelen, "Dijet azimuthal correlations in p-p and p-Pb collisions at forward LHC calorimeters", JHEP 12 (2022) 131.

This shows the motivation of the author to work on scientific problems that are not directly related to the thesis which, in my opinion, a very strong indicator of the excellent level of scientific integrity of the author.

The thesis manuscript is organized in a clear and logical way. The author first covers the basics in the introduction in Chapter 1. Then, the next two chapters (Chapter 2 and 3) is devoted to the analysis and computation of tree level actions, which is followed by two chapters (Chapter 4 and 5) that are devoted to computation and discussions of the quantum corrections. These four chapters include original results. The author presents a concise summary of the results in the Conclusions (Chapter 6) which is followed by an Outlook (Chapter 7) where he briefly discusses the further possible extensions of the obtained results. As stated earlier, even though the work presented in the thesis is highly technical, clear presentation of the results along with the detailed computations provided in the Appendix, help the reader to understand and reproduce the obtained results. The discussion and the bibliography shows that the author is well aware of the literature. The list of 101 references is well chosen and covers the literature related to the presented work. The thesis is written in good and clear English language with very high editorial quality. It is remarkable to see almost no typos in such a long text which indicates the high level of presentation. The only typo I spotted is "on-loop" which should be one-loop on page 6 in the third paragraph.

In conclusion, the doctoral thesis of mr. Hiren Kakkad is of high scientific relevance and of high quality. The obtained results are novel and significantly contributes to the development of the field. The presentation of the thesis is excellent. **The thesis definitely meets the international standards and fulfills Polish statutory requirements. Thus, I recommend that Mr. Hiren Kakkad to be admitted to further steps of the procedure of awarding of PhD in physics**.

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